

# Performance of Spatial Multiplexing by Transmit Antenna Selection using Singular Value Decomposition

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**Abstract:** In this paper, we analyze the performance of proposed diversity order of the optimal Transmit Antenna Selection (TAS) for Spatial Multiplexing (SM) systems with linear receivers. By using the diversity order  $N (M - L + 1)$  for optimal TAS by selecting  $L$  antennas among  $M$  transmit antennas in the SM system with  $M$  transmit antennas and  $N$  receive antennas, which coincides with the well-known results of the respective diversity orders  $MN$  and  $N$  for the special cases when  $L = 1$  and  $L = M$ .

**Keywords:** Diversity order, maximum likelihood (ML), minimum-mean-square-error (MMSE), spatial multiplexing(SM), transmit antenna selection (TAS).

## I. INTRODUCTION

In recent years, the remarkable ability of multiple inputs multiple-output (MIMO) wireless communication systems to provide spatial diversity or multiplexing gains has been clearly demonstrated. Transmit or receive diversity is a means to counter fading. In a MIMO channel fading may be favourable, through enlarging the degrees of freedom available for wireless communication [2]-[1]. By conveying maverick information streams in parallel through the Spatial channels, the data rate can be expanded. This effect is termed as spatial multiplexing [5]. Due to the hardware cost of multiple radiofrequency (RF) chains, effective antennas election schemes, such as Single transmit antenna selection(TAS) and TAS with space-time) and TAS with space-time codes, have been extensively studied [1]-[4]. In addition, for spatial multiplexing (SM) systems, many researchers have studied TAS, which simultaneously obtains high spectral efficiency and link reliability with relatively small hardware cost of multiple RF chain [5]-[8].

In [5], Heath and Paul raj presented TAS schemes for SM-Systems in terms of the minimum error rate when maximum-likelihood (ML) detection, zero-forcing, or a minimum-mean-square-error linear receiver was applied. In [6], multimode antenna selection schemes are proposed with linear receivers by considering all possible modes in multiplexing and selection diversity. The diversity order is achieved by antenna selection in spatial multiplexing(SM) systems. The diversity order of TAS for separately encoded SM systems with linear and decision-feedback receivers was analysed in [7]- [8]. Even if ML detection is difficult to use in practice, it is worth studying since ML detection achieves the best system performance. In addition, we observe that the existing low-complexity sphere decoding [9]-[10] is able to achieve the same result as ML detection, which makes TAS reach the best performance with a relatively low complexity decoder. In this paper, we

analyse the performance of optimal diversity order  $N (M - L + 1)$  of the TAS scheme in SM systems with ML and MMSE detection. By selecting  $L$  antennas among  $M$  transmit antennas in the SM system with  $M$  transmit antennas and  $N$  receive antenna.

## II. SYSTEM MODEL AND TRANSMIT ANTENNA SELECTION

We consider a frequency flat quasi-static fading channel with  $M$  transmit antennas and  $N$  receive antennas,

$$H = [h_1, h_2 \dots h_m] \sim CN(0, I_{NM})$$

$I_{NM}$  Denotes the  $N \times M$  matrix

where  $h_1, \dots, h_m$ , are the column vectors corresponding to each transmit antenna in Channel matrix  $H$ . Matrix  $H$  is independent and identically distribute circularly symmetric Gaussian random variables with zero mean and variance  $\sigma^2$ .

We assume that channel matrix  $H$  is perfectly known at the receiver rather than at the transmitter. Information that indicates the optimal  $L$  antennas among  $M$  transmit antennas is feed back to the transmitter. This setup network is analysed further.

In addition, we do not take into account channel estimation errors, i.e., error in the feedback path, and delay in the feedback path as in [5] and [7]. We apply SM to the transmitter by using selected  $L$  transmit antennas.

Then, there are

$$N_s = \binom{M}{L} = M/L (M - L) \text{ possible antenna}$$

Subsets defined as

$$S_1 = \{h_1, h_2 \dots h_{l-1}, h_l\}$$

$$S_2 = \{h_1, h_2 \dots h_{l-1}, h_l\}$$

$$S_{Ns} = \{Zh_{m-1+1}h_{m-1+2}, h_{m-1}, h_m\} \dots (1)$$

Where  $L$  column vectors in  $S_j$  compose channel matrix  $H_j$ . Therefore, in the case that  $S_j$  is selected, the  $N \times 1$  received signal vector in the SM system can be written as  $y = H_j x + n$  ..... (2)

Where  $n \sim CN(0, \sigma^2 I_N)$  is the noise vector,  $x = [x_1, x_2, \dots, x_L]^T \in A^L$  is the transmitted signal vector with  $E\{|x_i|^2\} = 1/L$ , Where  $E\{\cdot\}$  denotes the expectation with respect to random variable and  $A$  is the set of message symbols from the Mc-array signal constellation. We remark that the average signal to-noise ratio (SNR) is  $SNR = 1/\sigma^2$ .

A. Maximum likelihood [ml] detection

ML detection is optimal in the sense of minimum error probability when all data vectors are equally acceptable, and it fully exploits the available diversity. For the given system, the ML detection of the transmitted signal vector is

$$\hat{x}_{ML} = \underset{x \in A^L}{\operatorname{argmin}} \|y - H_j x\|^2 \dots (3)$$

Where  $\|\cdot\|$  represent the Frobenius norm of a matrix and  $(\cdot)$  is the signal estimate, to determine the performance of ML detection, we would like to compute the bit and symbol error probabilities [11, 12]. As the worst-case scenario in [5], the minimum square distance defined as

$$\Gamma_j = \min_{x, x \neq \check{x}} \|H_j(x - \check{x})\|^2 \dots (4)$$

Determines the performance of ML detection. Therefore, as shown in [5] and [13], the optimal TAS for ML detection with respect to the diversity order is selecting  $L$  transmit antennas in the  $j^{\text{th}}$  antenna set  $S_j$ , which satisfies

$$\hat{j} = \underset{j \in \{1, \dots, N_s\}}{\operatorname{argmin}} \Gamma_j \quad (5)$$

Letting  $H_j = [h_{j1}, \dots, h_{jl}]$  and

$$h_{j1} = [h_{1,j1} \dots h_{N,j1}]^T,$$

$l = 1, \dots, L$ ,  $\Gamma_j$  in (4) can be rewritten as

$$\min_{x, x \neq \check{x}} \|h_{j1}(x - \check{x}) + \dots + h_{jl}(x - \check{x})\|^2$$

Since we consider an arbitrary order of  $j_1, \dots, j_l$ , there are many kinds of values of  $\Gamma_j$ . We point out that even if the column vectors in  $H_j$  are reordered, the values of  $\Gamma_j$  can be considered to be equal by the same modulation of  $x_1, \dots, x_L$ . Thus, we assume that the index order of the column vectors in  $H_j$  is increased, i.e.,  $1 \leq j_1 < \dots < j_l \leq M$ .

B. Minimum mean-square error [MMSE -SIC] detection

In linear equalization based detection, an estimate of the transmitted signal vector  $x$  is formed as

$$r = Gy \dots (6)$$

Where 'G' is the equalization matrix, by using equation (2)

Above relation can be written as,  $r = GH_j x + Gn$ . The detected signal vector is then obtained as  $\hat{x} = Q\{r\}$  where  $Q\{\cdot\}$  denotes element wise quantization proportion to the symbol alphabet  $A$ . In Minimum mean-square error

(MMSE) equalizer,  $G$  minimizing the mean-square error  $E\{\|Gy - x\|^2\}$  [23]; thus, result of MMSE equalization is  $Y_{mmse} = Gy$  With the distance  $\|y - H_j x\|^2$  augmented by a penalty term  $\sigma^2 \|r\|^2$  that prevents  $r$  from growing too large [22];

III. PROPOSED DIVERSITY ANALYSIS WITH OPTIMAL DETECTOR

Here, we are using the diversity order for the SM [16], the lower bound on diversity is  $d_{TAS} \geq N(M - L + 1)$  and upper bound on diversity is  $d_{TAS} \leq N(M - L + 1)$  of the proposed diversity are the same. Therefore, we conclude that the achievable diversity order in the SM system with the optimal TAS is

$$d_{TAS} = N(M - L + 1)$$

Diversity order [7] for an  $N \times M$  for SM system employing linear receivers ZF and MMSE. An upper bound,  $(M - L + 1)(N - 1)$  and a lower bound  $(M - L + 1)(N - L + 1)$  for general  $L$  is achieved where 1 For example taking  $M=5$ ,  $L=2$  and  $N=2$ . The performance of the proposed diversity is given by calculating the achievable diversity order in the SM system with the optimal TAS is  $d_{TAS} = N(M - L + 1)$ . Then  $2(5-2+1) = 8$  while for diversity order [7] for upper bound,  $(5-2+1)(2-1) = 4$  and for lower bound,  $(5-2+1)(2-2+1) = 4$ . We see that the diversity orders for upper and lower bound are same. The larger diversity order, the probability is lesser that all these channels fade concurrently, and thus the reliability of data detection can be improved. If the available diversity is  $d_{TAS}$ , the symbol error rate (SER) of the optimal maximum likelihood (ML) detector decays like  $SNR^{-d_{TAS}}$  in the high-SNR regime [24, 19].

In general if the SER of some detector decays like  $SNR^{-\gamma}$ ,  $\gamma = \max d_{TAS}$  we say that the detector can utilize  $\gamma^{\text{th}}$ -order diversity. The ML detector is optimal and fully utilize the available diversity. Unfortunately, the computational complexity of a direct implementation of the ML detector expand exponentially with the number of transmit antennas  $M$ , and it may be too high already for moderate system and constellation sizes [17]. Several efficient suboptimal detection techniques have therefore been proposed or adapted from the field of multiuser detection [18,21]. Whereas these techniques are much less computationally demanding than the ML detector, they are usually unable to utilize a large part of the available diversity, and thus their performance tends to be appreciably inferior than that of ML detection.

IV. SIMULATION RESULTS

In this section, the simulations studies have been carried out using MATLAB software and compare these results with simulation results of the novel spatial modulation using MIMO spatial multiplexing [35]. In this thesis the BER values have been computed as a function of SNR for different modulation and different combinations of SM-MIMO systems using Minimum Mean Square Error-Signal interference cancellation (MMSE-SIC) and

Maximum Likelihood (ML) equalizers. In MATLAB simulation parameters, M denotes the total number of antennas at the transmitter side, L denotes the selected active transmit antenna at the transmitter side and N denotes the total number of receiving antennas at the receiver side. We assume a Rayleigh flat fading channel, and the channel is perfectly notable to the receiver. The noise on severally receive antenna is assumed to be AWGN (Additive white Gaussian noise).

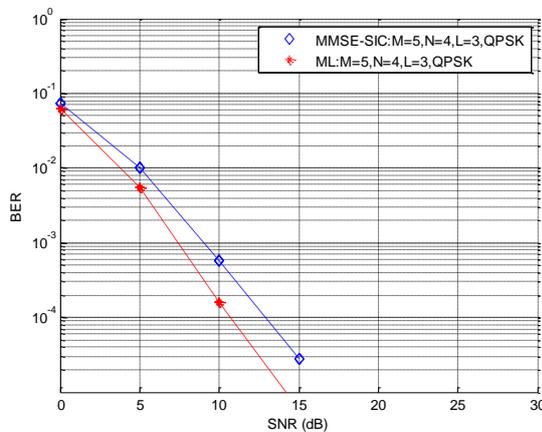


Figure 5.1 M=5, N=4, L=3, QPSK modulation with ML & MMSE-SIC equalizer

In Figure 5.1 we plot the BER performance of the 6 bps/Hz spectral efficiency with QPSK modulation. From the graphical analyses, it is evident that BER tends to decrease dramatically for ML equalizer compared to the MMSE-SIC equalizers.

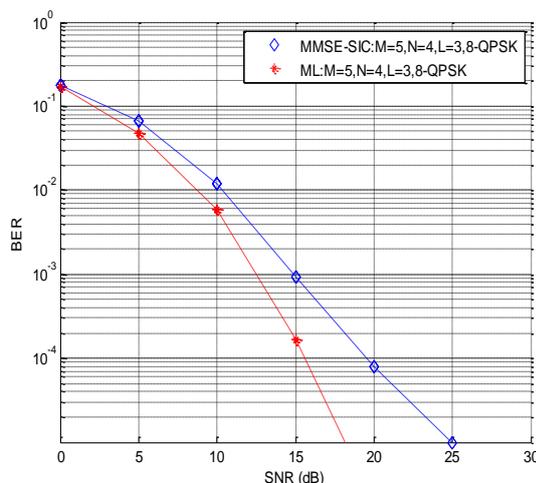


Figure 5.2 M=5, N=4, L=3, 8-QPSK modulation with ML & MMSE-SIC equalizer

In Figure 5.2 we plot the BER performance of the 9 bps/Hz spectral efficiency with 8-QPSK modulation. From the graphical persual, it is obvious that BER tends to decrease dramatically for ML equalizer compared to the MMSE-SIC equalizers. But if we compare this simulation results with simulation results of the Figure 5.1, then we can say as the modulation order expanded then the BER performance decreases.

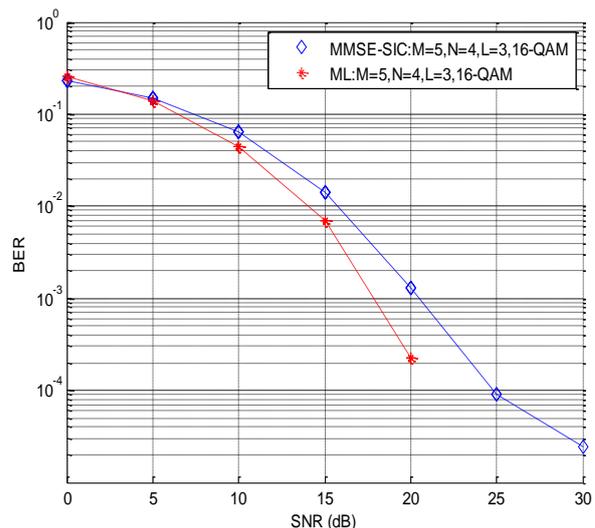


Figure 5.3 M=5, N=4, L=3, 16-QAM modulation with ML & MMSE-SIC equalizer

In Figure 5.3 we plot the BER performance of the 12 bps/Hz spectral efficiency with 16-QAM modulation. From the graphical analyses, it is evident that BER tends to decrease dramatically for ML equalizer compared to the MMSE-SIC equalizers. But if we compare this simulation results with simulation results of the Figure 5.2 and 5.1, then we can say as the modulation order increases then the BER performance decreases. Thus the BER improvement is directly related with the lower modulation order.

## V. CONCLUSION

In this paper, we have assess the performance of the proposed diversity order of the optimal TAS scheme for SM systems with ML, MMSE detector. BER decreases as the diversity order increases. By exploiting optimal diversity–multiplexing trade-off such diversity in SM system can be tremendous for downlink reliable high-data-rate communications, where large number of transmit antennas at the base stations but few receive antennas at the mobiles. The performance upgrade of the proposed scheme over optimal SM and V-BLAST increases as the diversity increases, which makes it useful for high data rate transmission systems e.g. WiMAX and LTE-Advanced.

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